COMMON CORE-ALIGNED TASK
WITH INSTRUCTIONAL SUPPORTS

Mathematics

GRADE 5 MATH: TIME FOR RECESS

UNIT OVERVIEW

This packet contains a curriculum-embedded CCLS aligned task and instructional supports. The task, Time for Recess, has been designed to give a thorough understanding of multiplication of fractions, using both an area model and the algorithm.

TASK DETAILS

Task Name: Time for Recess

Grade: 5

Subject: Math

Depth of Knowledge: 5

Task Description: After an in-depth unit of study involving multiplication of fractions through modeling and use of the standard algorithm, students will complete three real-world problems to demonstrate an understanding of interpreting a fraction as division of the numerator by the denominator and multiplying fractions using an area model and the standard algorithm.

Standards:
5.NF.3—Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

5.NF.4a—Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. Interpret the product $(a/b) \times q$ as a part of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$.

5.NF.4b—Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

Standards for Mathematical Practice:
MP.1—Make sense of problems and persevere in solving them.
MP.3—Construct viable arguments and critique the reasoning of others.
MP.6—Attend to precision.

NYC Department of Education
Dennis M. Walcott, Chancellor

TABLE OF CONTENTS

The task and instructional supports in the following pages are designed to help educators understand and implement tasks that are embedded in Common Core-aligned curricula. While the focus for the 2012–2013 Instructional Expectations is on engaging students in Common Core-aligned culminating tasks, it is imperative that the tasks are embedded in units of study that are also aligned to the new standards. Rather than asking teachers to introduce a task into the semester without context, this work is intended to encourage analysis of student and teacher work to understand what alignment looks like. We have learned through the 2010–2011 Common Core pilots that beginning with rigorous assessments drives significant shifts in curriculum and pedagogy. Universal Design for Learning (UDL) support is included to ensure multiple entry points for all learners, including students with disabilities and English language learners.

Acknowledgements: This task and unit were developed by Jason Westerlund, NYCDOE Math Common Core Fellow and Grade 5 teacher; Jeff Moss, teacher; Margot Sawicki, teacher; Matthew Schneider, teacher, P.S. 101, The School in the Gardens, Queens, CFN 207.

GRADE 5 MATH: TIME FOR RECESS

PERFORMANCE TASK

NYC Department of Education
Dennis M. Walcott, Chancellor

TIME FOR RECESS AND SOME LUNCH

1. The 4th and 5th graders both have outdoor recess at the same time. The 4th graders like to play basketball, the 5th graders like to play soccer, and some kids from both grades like to sit and read. In order to keep everyone safe, the school sets aside 1/5 of the playground for kids to sit and read. Of the remaining playground area, the school sets aside 2/3 for students to play soccer. Use the diagram below to show what fraction of the entire playground is set aside for soccer. Defend your diagram using a mathematical equation and written explanation.

PLAYGROUND

TIME FOR RECESS (cont'd.)

2. After playing, the students go inside for lunch. One student brings 3 boxes of cookies to split between himself and 14 of his closest friends. If each student gets the same amount, what fraction of a box of cookies does each student receive? Explain your thinking using a number sentence, model, and written explanation.

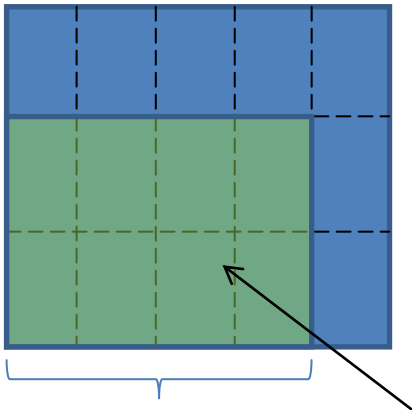
3. If each box contains 20 cookies, and one student asks for 5 cookies, is that fair? Why or why not? Use mathematical thinking to defend your answer.

4. Please find the area of a rectangle with a length of 3/7 inch and a width of 2/3 inch below using both a model and a number sentence. Explain where your number sentence is represented in your model.

TIME FOR RECESS AND SOME LUNCH ANSWER KEY

1. The 4th and 5th graders both have outdoor recess at the same time. The 4th graders like to play basketball, the 5th graders like to play soccer, and some kids from both grades like to sit and read. In order to keep everyone safe, the school sets aside 1/5 of the playground for kids to sit and read. Of the remaining playground area, the school sets aside 2/3 for students to play soccer. Use the model below to show what fraction of the entire playground is set aside for soccer. Defend your diagram using a mathematical equation and written explanation.

PLAYGROUND



2/3 of the yard.

4/5 of the yard remaining after 1/5 is used for students to read.

8/15 of the yard is the area remaining for students to play soccer.

ANSWER: The diagram at the left was tiled appropriately. The shaded region is clearly 8/15.

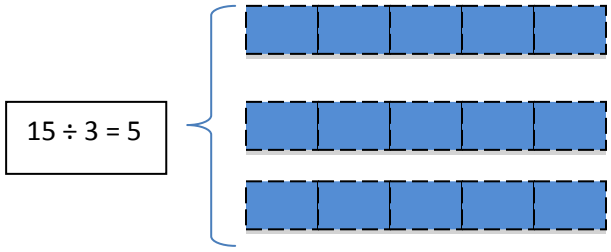
The correct number sentence is:

4/5 X 2/3 = 8/15

The written explanation should include how the student tiled the model, why they shaded the region they shaded, and how it relates to the number sentence.

TIME FOR RECESS (cont'd.)

2. After playing, the students go inside for lunch. One student brings 3 boxes of cookies to split between himself and 14 of his closest friends. If each student gets the same amount, what fraction of a box of cookies does each student receive? Explain your thinking using a number sentence, model, and written explanation.



ANSWER:

The correct number sentence is:

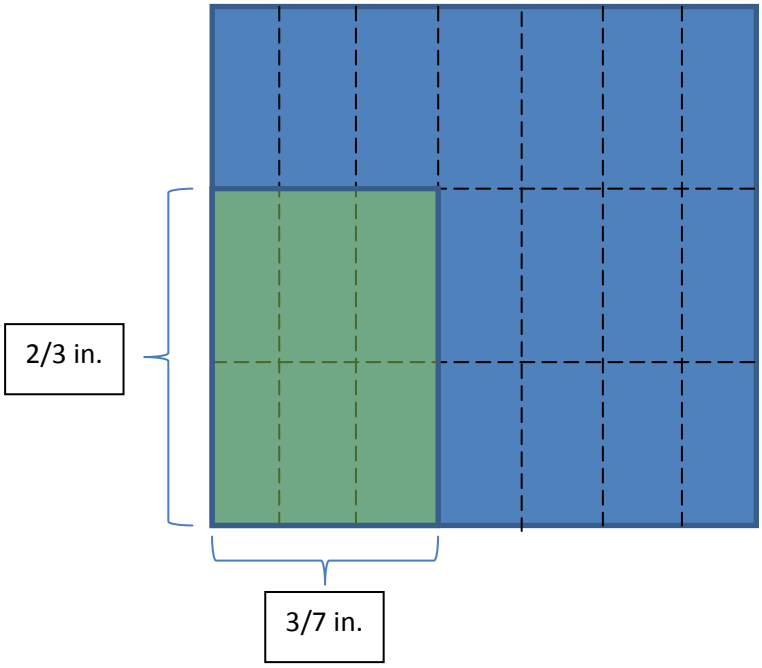
1 + 14 = 15 → 15 ÷ 3 = 5

3 boxes divided by 15 people. As a fraction that would be 3/15. Simplified it would be 1/5. I know my answer is correct because a fraction is really division of the numerator by the denominator.

3. If each box contains 20 cookies, and one student asks for 5 cookies, is that fair? Why or why not? Use mathematical thinking to defend your answer.

ANSWER: No, it is not fair, because if the cookies are split evenly each person will get 4 cookies. If one student gets 5, you can't split the cookies evenly. If each person gets 1/5 of a box of cookies, and there is 20 cookies in each box, I know that each student gets 1/5 of 20. I know that I can write that as 1/5 X 20. 1/5 X 20 = 20/5. Simplified, 20/5 is equal to 4.

4. Please find the area of rectangle with a length of 3/7 inch and a width of 2/3 inch below using both a model and a number sentence. Explain where your number sentence is represented in your model.



Number Sentence:

A = L X W
A = 3/7 in. X 2/3 in.
A = 6/21 sq. in.
A = 2/7 sq. in.

Answer: The area of a rectangle with a length of 3/7 inch and a width of 2/3 inch is 6/21, or 2/7 square inches when simplified. I proved this by tiling a model and using the area formula. Both my diagram and number sentence match. I've indicated the two numbers I was multiplying as my fractional sides in my diagram. The area 2/7 square inches is the shaded portion of my diagram.

GRADE 5 MATH: TIME FOR RECESS

RUBRIC

The rubric that follows can be used to assess student mastery of the content and performance standards in the Time for Recess task. There are four levels of assessment: novice, apprentice, practioner, and expert. The standards for each question on the task are also listed in the rubric.

Time for Recess (Multiplying Fractions) Task Rubric

Question	Standard	Novice	Apprentice	Practitioner	Expert
#2	5.NF.3—Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem.	The student has limited or no evidence demonstrating the ability to solve word problems by interpreting a fraction as division of the numerator by the denominator.	The student solves word problems by interpreting a fraction as division of the numerator by the denominator with some inaccuracies, and/or with an almost complete explanation.	The student solves word problems by interpreting a fraction as division of the numerator by the denominator and expresses this understanding with some clarity, using some elements of modeling, mathematical statements, and a written explanation.	The student consistently solves word problems by interpreting a fraction as division of the numerator by the denominator and expresses this understanding clearly through mathematical statements, models, and written explanation.
#4, #3	5.NF.4a—Interpret the product $(a/b) \times q$ as a part of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. For example, use a visual fraction model to show $(2/3) \times 4 = 8/3$, and create a story context for this equation. Do the same with $(2/3) \times (4/5) = 8/15$. (In general, $(a/b) \times (c/d) = ac/bd$.)	The student cannot identify or apply the correct standard algorithm for multiplying fractions by fractions.	The student identifies the correct standard algorithm for multiplying fractions by fractions, but has inaccuracies when carrying it out that show misunderstandings of the concept.	The student applies the correct standard algorithm for multiplying fractions by fractions with some inaccuracies that don't take away from understanding of the concept.	The student consistently applies the correct standard algorithm for multiplying fractions by fractions and connects the algorithm to an appropriate model.
#1, #4	5.NF.4b—Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.	The student cannot find the area of a rectangle with fractional sides by tiling with appropriate unit squares or applying the area formula.	The student finds an incorrect area of a rectangle with fractional sides by tiling with appropriate unit squares and applying the area formula, but shows some evidence of understanding of the concept.	The student finds the area of a rectangle with fractional sides by tiling with appropriate unit squares and applying the area formula with some inaccuracies that don't take away from understanding of the concept.	The student consistently finds the area of a rectangle with fractional sides by tiling with appropriate unit squares and applying the area formula.

GRADE 5 MATH: TIME FOR RECESS

ANNOTATED STUDENT WORK

This section contains annotated student work at a range of score points. The student work shows examples of student understandings and misunderstandings of the task.

Grade 5 Math: Time for Recess
Annotated Student Work: Level 1

Student 1: Annotated Student Work Summary

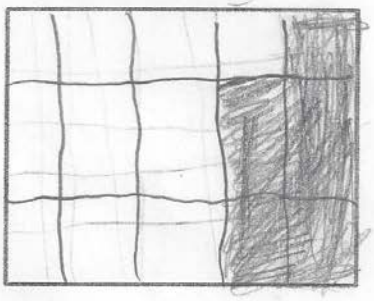
Overall, this student performed at Level One. The student demonstrates only minor fractional understanding. They do not recognize fractions as division of the numerator by the denominator (5.NF.3). They do not demonstrate major understanding of the concept of finding a fraction of a fraction by multiplying or using a fraction model (5.NF.4a). They do not demonstrate major understanding of the concept of area of a rectangle by multiplying fractional side lengths or tiling (5.NF.4b). The student needs to revisit these concepts, both to improve the use of modeling, as well as to improve the understanding of the basic algorithm for multiplying fractions.

Level 1 Sample:

TIME FOR RECESS & SOME LUNCH

1. The fifth graders and fourth graders both have outdoor recess at the same time. The fourth graders like to play basketball, the fifth graders like to play soccer, and some kids from both grades like to sit and read. In order to keep everyone safe, the school sets aside 1/5 of the playground for kids to sit and read. Of the remaining playground area, the school sets aside 2/3 for students to play soccer. Use the model below to show what fraction of the entire playground is set aside for soccer. Defend your model using a mathematical equation, and written explanation.

PLAYGROUND



I think that $\frac{3}{3}$ of $\frac{1}{5}$ of the playground was set for Soccer.

Teacher Note: Student demonstrates confusion of fractional concepts. He attempts to tile the rectangle, and recognizes fifths and third. However, the tiles are not uniform in size, and the student does not apply the tiling concept to find an area of a rectangle with fractional sides or a fraction "of" a fraction (5.NF.4a, b).

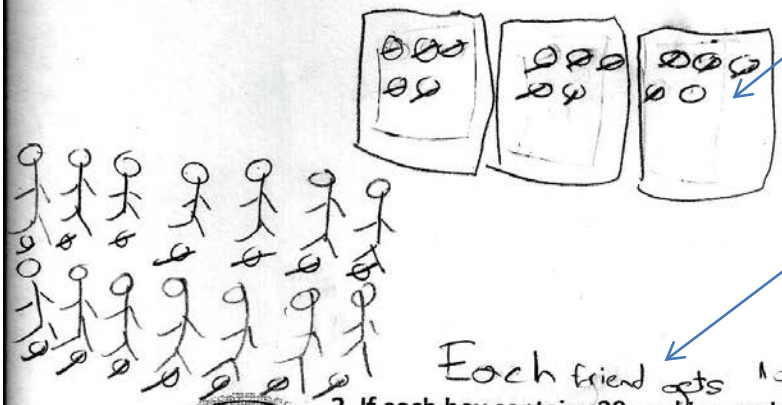
Teacher Note: The student does not demonstrate the ability to apply the standard algorithm for multiplying fractions (5.NF.4a).

Teacher Note: The student answer is inaccurate. The student misunderstands the problem, uses the wrong fractions, and does not complete the work.

Grade 5 Math: Time for Recess
Annotated Student Work: Level 1

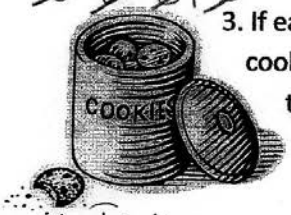
TIME FOR RECESS (cont'd.)

2. After playing, the students go inside for lunch. One student brings 3 boxes of cookies to split between **himself and 14** of his closest friends. If each student gets the same amount, what fraction of a box of cookies does each student receive? Explain your thinking.

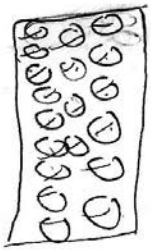


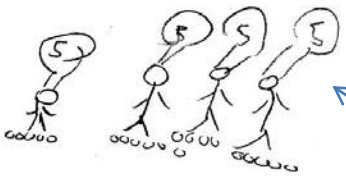
Each friend gets 1 and a half

3. If each box contains 20 cookies, and one student asks for 5 cookies is that fair? Why or why not? Use mathematical thinking to defend your answer.



If there is 3 more students and they want some cookies yes because each get five cookies.





Teacher Note: The Student shows an attempt at fractional understanding by drawing 15 circles and breaking them into 3 rectangles. However, the student does not demonstrate understanding of fractions as division of the numerator by the denominator. The answer of 1 and a half is also incorrect (5.NF.3).

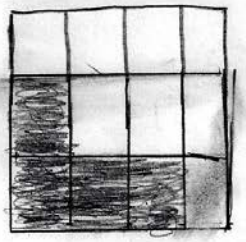
Teacher Note: Student explanation that including 3 more students in this situation would change the answer is incorrect and unsupported.

Teacher Note: The student demonstrates an understanding of division, but does not relate this understanding to fractional concepts (5.NF.3) and cannot create a viable argument or critique the reasoning of others (MP.3).

Grade 5 Math: Time for Recess
Annotated Student Work: Level 1

4. Please find the area of a rectangle with a length of 3/7 in. and a width of 2/3 in. below using both a model and a number sentence. Explain where your number sentence is represented in your model:

Teacher Note: The student demonstrates some fractional understanding by breaking up the rectangle horizontally into thirds and shading 2/3. However, the student does not break up the shape accurately into sevenths vertically. Instead, the student confuses 7ths and 4ths and attempts to solve the problem using addition as opposed to multiplication. The student is not able to accurately tile the rectangle or apply the area formula (5.NF.4b).



$\frac{2}{3}$ $\frac{3}{7}$

$\frac{2}{4}$ $\frac{3}{4}$ $\frac{6}{4} = 1\frac{2}{4}$

two of the rectangle is $\frac{6}{4}$ and in simpest from $1\frac{2}{4}$

$\frac{1}{2}$: 1, 2, 4

$\frac{3}{4}$: 1, 4

Grade 5 Math: Time for Recess
Annotated Student Work: Level 2

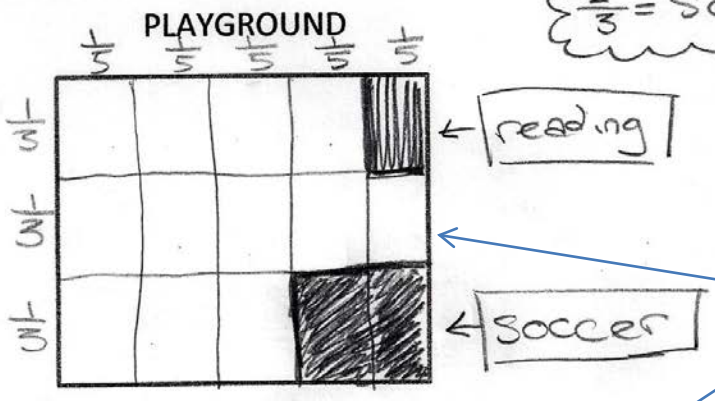
Student 2: Annotated Student Work Summary

Overall, this student performed at Level Two. The student demonstrates some fractional understanding. They recognize fractions as division of the numerator by the denominator, but have some inaccuracies when tackling problems involving this understanding (5.NF.3). They can create fractional models that are tiled correctly, but are missing the connections needed to shade the appropriate tiles (5.NF.4a). They demonstrate some understanding of the concept of finding the area of a rectangle by multiplying fractional side lengths or tiling (5.NF.4b). They tile correctly, but once again are not making the connections needed to shade the appropriate amount. They also use the wrong operation (addition) when applying the area formula. The student needs to revisit these concepts, both to improve the application of models, and to build a better understanding of the area formula.

TIME FOR RECESS & SOME LUNCH

1. The fifth graders and fourth graders both have outdoor recess at the same time. The fourth graders like to play basketball, the fifth graders like to play soccer, and some kids from both grades like to sit and read. In order to keep everyone safe, the school sets aside 1/5 of the playground for kids to sit and read. Of the remaining playground area, the school sets aside 2/3 for students to play soccer. Use the model below to show what fraction of the entire playground is set aside for soccer. Defend your model using a mathematical equation, and written explanation.

$\frac{1}{5}$ = read
$\frac{2}{3}$ = soccer

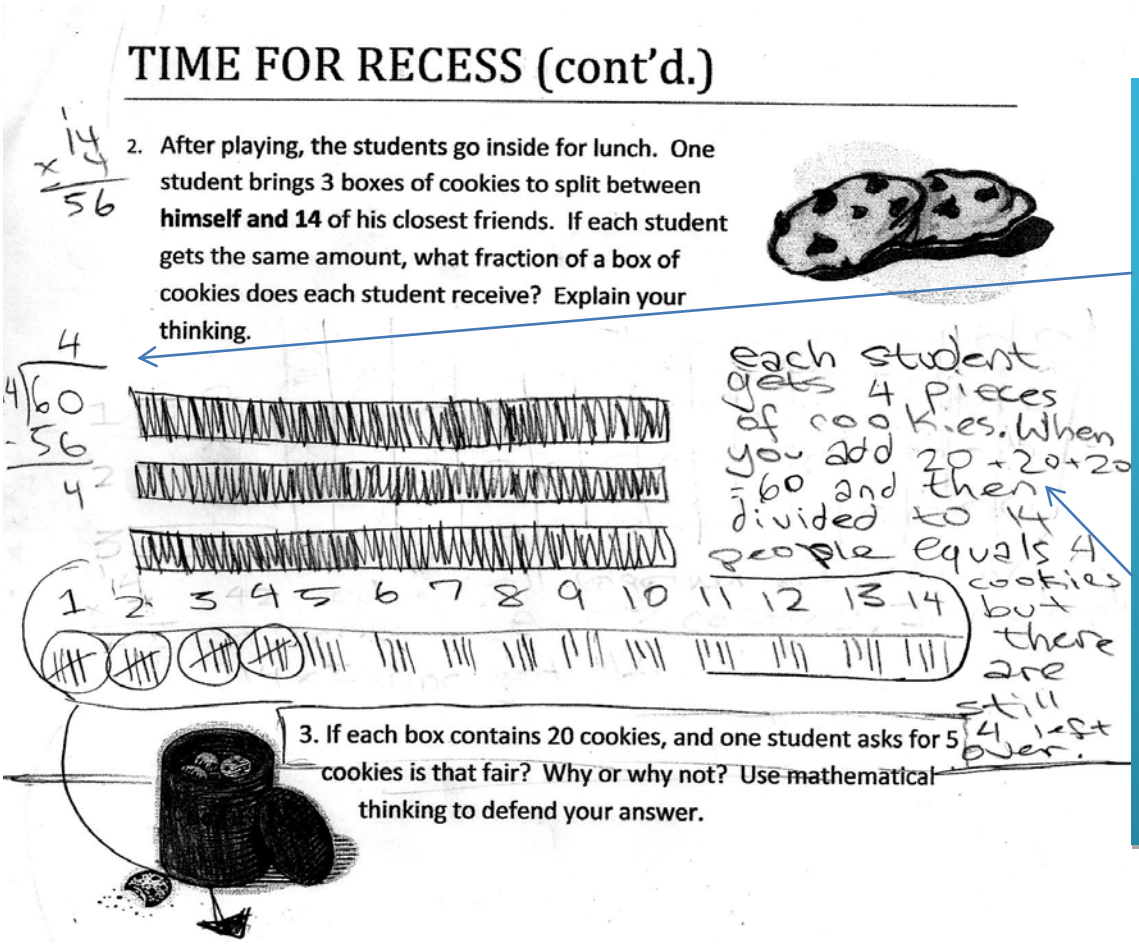


$\frac{2}{14} = \frac{1}{7}$

$\frac{1}{7}$ of the playground that was set aside was taken up by the soccer area.

Teacher Note: The student finds and incorrect area of a rectangle with fractional sides by tiling with appropriate unit squares (5.NF.4a & b). Although the student broke down the shape into appropriate tiles, the student shades an incorrect portion of the rectangle, finding an inaccurate area.

Grade 5 Math: Time for Recess
Annotated Student Work: Level 2



TIME FOR RECESS (cont'd.)

14 × 4 = 56

2. After playing, the students go inside for lunch. One student brings 3 boxes of cookies to split between **himself and 14** of his closest friends. If each student gets the same amount, what fraction of a box of cookies does each student receive? Explain your thinking.

4
14 | 60
-56
4

each student gets 4 pieces of cookies. When you add 20+20+20 (60 and then divided to 14 people equals 4 cookies but there are still 4 left over.

1 2 3 4 5 6 7 8 9 10 11 12 13 14

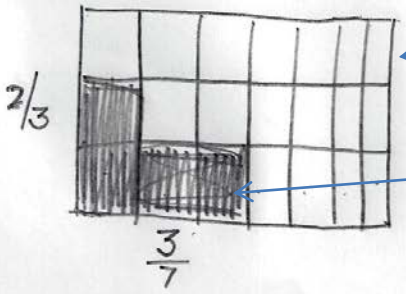
3. If each box contains 20 cookies, and one student asks for 5 cookies is that fair? Why or why not? Use mathematical thinking to defend your answer.

It wouldn't be fair if only one person got the 5th cookie but it still wouldn't be fair if you give him or her 1 more cookie even though there is still 4 more cookies left.

Teacher Note: The student is able to recognize a fraction as division of the numerator by the denominator (5.NF.3) with some inaccuracies. The student uses a numerator of 60 (taken by multiplying the 3 boxes times the 20 cookies from problem 3) and a denominator of 14 instead of 15 to find an incorrect answer.

Teacher Note: The student reaches a correct answer, but does not use accurate mathematical evidence to create a viable argument, or critique the reasoning of others (MP.3).

4. Please find the area of a rectangle with a length of 3/7 in. and a width of 2/3 in. below using both a model and a number sentence. Explain where your number sentence is represented in your model:



$\frac{2}{3}(\frac{14}{21})$

$\times \frac{3}{7}(\frac{9}{21})$

$\frac{25}{21}$ | $1\frac{4}{21}$

Teacher Note: The student is able to accurate tile a rectangle with fractional side lengths (5.NF.4b), but does not apply appropriate shading to accurately find the area. The student indicates understanding of the area formula, by setting up the appropriate algorithm, but then uses addition instead of multiplication. The student also records their answer incorrectly in their explanation.

the area of the whole rectangle is $1\frac{4}{21}$ because I multiplied $\frac{2}{3} \times \frac{3}{7} = \frac{21}{25}$ and then I changed it into a mixed number.

Grade 5 Math: Time for Recess
Annotated Student Work: Level 3

Student 3: Annotated Student Work Summary

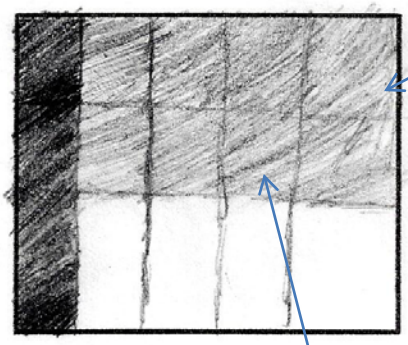
Overall, this student performed at Level Three. The student demonstrates sound fractional understanding. They recognize fractions as division of the numerator by the denominator, but have some minor inaccuracies when tackling problems involving this understanding (5.NF.3). They can create fractional models that are tiled correctly, and apply the correct algorithm to arrive at the correct answer when multiplying fractions, but have minor inaccuracies in their work, or their explanation is incomplete (5.NF.4a). They demonstrate sound understanding of the concept of finding the area of a rectangle by multiplying fractional side lengths and tiling (5.NF.4b). They tile correctly, and apply the area formula, but their written explanation is the missing component that would tie the two concepts together. The student needs to work on fluency and accuracy in their details, as well as improving their written explanations.

Level 3 Student Sample

TIME FOR RECESS & SOME LUNCH

1. The fifth graders and fourth graders both have outdoor recess at the same time. The fourth graders like to play basketball, the fifth graders like to play soccer, and some kids from both grades like to sit and read. In order to keep everyone safe, the school sets aside 1/5 of the playground for kids to sit and read. Of the remaining playground area, the school sets aside 2/3 for students to play soccer. Use the model below to show what fraction of the entire playground is set aside for soccer. Defend your model using a mathematical equation, and written explanation.

PLAYGROUND



Reading
Soccer

□ = 8/15 (2/3)

or 2/3 × 1/5 = 2/15

Denominator

8/15 is the answer because if you split up the whole area, and shade in the designated area for soccer, 8/15 is shaded in.

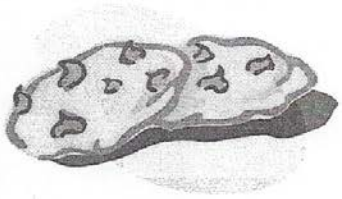
Teacher Note: Student uses a visual fraction model to find the fraction of a fraction, and tiles appropriately to find the area of a rectangle with fractional side lengths (5.NF.4a, b).

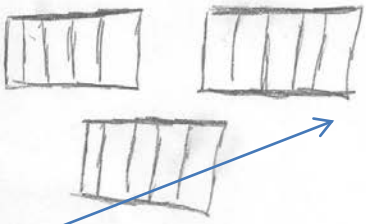
Teacher Note: Student uses an incorrect factor in multiplying side lengths (1/5 instead of 4/5), but the model and written explanation include the correct answer.

Grade 5 Math: Time for Recess
Annotated Student Work: Level 3

TIME FOR RECESS (cont'd.)

2. After playing, the students go inside for lunch. One student brings 3 boxes of cookies to split between **himself and 14** of his closest friends. If each student gets the same amount, what fraction of a box of cookies does each student receive? Explain your thinking.



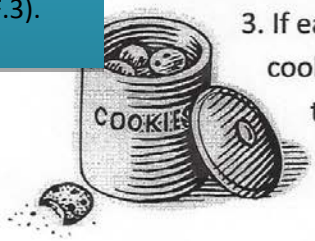


$3\overline{)15}$ 5

Each person gets 5 cookies

Teacher Note: The student creates accurate fraction models and uses division to find the answer, however he gets an incorrect answer of 5 cookies because he divides the denominator by the numerator (5.NF.3).

3. If each box contains 20 cookies, and one student asks for 5 cookies is that fair? Why or why not? Use mathematical thinking to defend your answer.



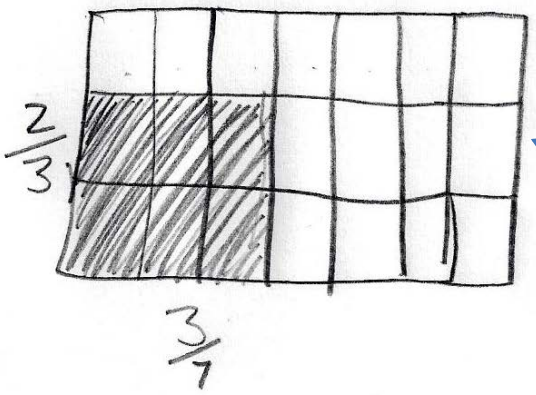
$5\overline{)60}$ 12 cookies in all

5
10
16
0

It isn't fair because if each person gets 5, only 12 people can have cookies.

Teacher Note: The student accurately divides the total number of cookies by 5 to reach a correct answer. He uses sound mathematical thinking to create a viable argument, and critique the reasoning of others (MP.3).

4. Please find the area of a rectangle with a length of 3/7 in. and a width of 2/3 in. below using both a model and a number sentence. Explain where your number sentence is represented in your model:



$\frac{2}{3} \times \frac{3}{7} = \frac{6}{8}$

Teacher Note: The student accurately finds the area of a rectangle with fractional sides both by tilting with appropriate unit tiles, and by using the area formula (5.NF. 4b). However, the student does not include an explanation that indicates the connections between the two.

Grade 5 Math: Time for Recess
Annotated Student Work: Level 4

Student 4: Annotated Student Work Summary

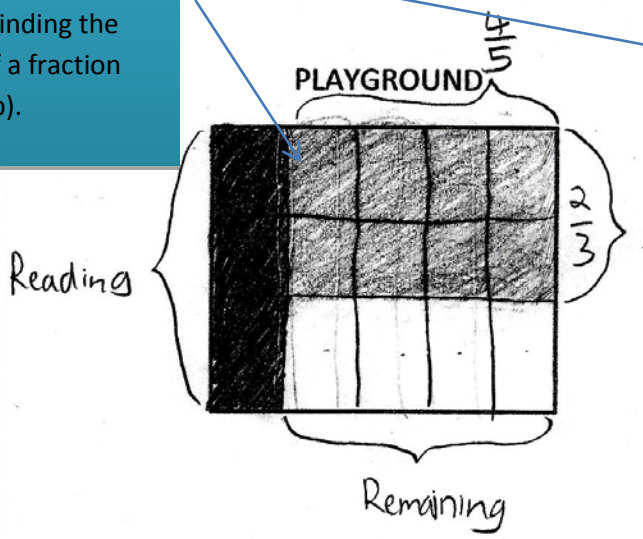
Overall, this student performed at Level Four. The student demonstrates consistent deep fractional understanding. They recognize fractions as division of the numerator by the denominator, and apply this understanding to solve word problems involving whole numbers that have fractional products (5.NF.3). They can create fractional models that are tiled correctly, and apply the correct algorithm to arrive at the correct answer when multiplying fractions accurately, with complete, clear explanations (5.NF.4a). They demonstrate consistent deep understanding of the concept of finding the area of a rectangle by multiplying fractional side lengths and tiling (5.NF.4b). They tile correctly, apply the area formula, and include a clear, detailed written explanation that ties the two concepts together. The student consistently exhibits a deep understanding of all content standards in this task.

Level 4 Student Sample

TIME FOR RECESS & SOME LUNCH

1. The fifth graders and fourth graders both have outdoor recess at the same time. The fourth graders like to play basketball, the fifth graders like to play soccer, and some kids from both grades like to sit and read. In order to keep everyone safe, the school sets aside 1/5 of the playground for kids to sit and read. Of the remaining playground area, the school sets aside 2/3 for students to play soccer. Use the model below to show what fraction of the entire playground is set aside for soccer. Defend your model using a mathematical equation, and written explanation.

Teacher Note: The student demonstrates the ability to find the area of a rectangle with fractional side lengths by tiling and by multiplying the side lengths in reference to a real-world problem involving finding the fraction of a fraction (5.NF.4a, b).



$\frac{4}{5} \times \frac{2}{3} = \boxed{\frac{8}{15}}$

$\frac{8}{15}$ of the playground is set aside for soccer. I know this because $\frac{1}{5}$ of the playground is for readers. To find what part of the playground is set aside for soccer, you'll have to forget about the $\frac{1}{5}$ part and shade in $\frac{2}{3}$. Finally, count the shaded boxes (besides the $\frac{1}{5}$) and then count the rest plus the amount you shaded.

Teacher Note: The student's written explanation is clear, detailed, and accurate, and demonstrates a strong understanding of multiplication of fractions (5.NF.3a, b).

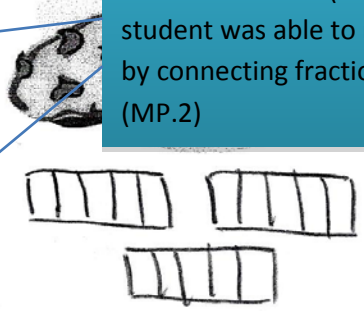
Grade 5 Math: Time for Recess
Annotated Student Work: Level 4

Teacher Note: The student interprets fraction as division of the numerator by the denominator, and solve word problems involving division of whole numbers leading to answer in the form of fractions (5.NF.3). The student was able to reason abstractly by connecting fractions to decimals. (MP.2)

TIME FOR RECESS (cont'd.)

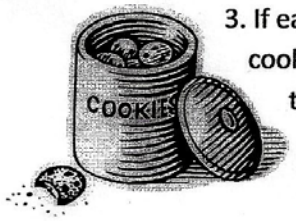
2. After playing, the students go inside for lunch. One student brings 3 boxes of cookies to split between **himself and 14** of his closest friends. If each student gets the same amount, what fraction of a box of cookies does each student receive? Explain your thinking.

0.2
15|3.0



15 ÷ 3 = 5 1 + 14 = 15

Each student will recive $\frac{1}{5}$ box of cookies. To find the answer, I first added the amount of people who were going to eat the cookies. The answer is 15. Since there is 3 boxes of cookies, you have to divide the amount of people by the boxes of cookies. The answer would



3. If each box contains 20 cookies, and one student asks for 5 cookies is that fair? Why or why not? Use mathematical thinking to defend your answer.

Teacher Note: The student advances their understanding of 5.NF.3 by creating a viable argument and supporting it with sound mathematical thinking by critiquing the reasoning of others (MP.3).

20 × 3 = 60

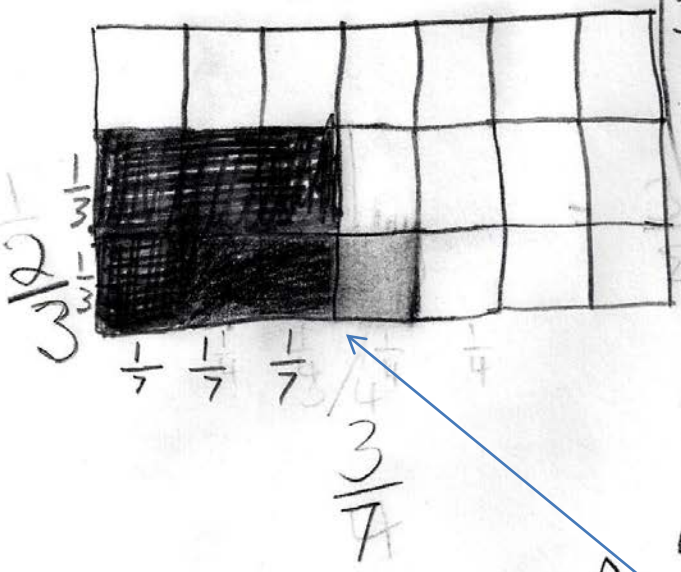
12
5|60
5
10

15 students

It won't be fair because there are 15 students. If each student got 5 cookies, there won't be enough cookies for everyone. Each student could get 4 cookies be 5.

Grade 5 Math: Time for Recess
Annotated Student Work: Level 4

4. Please find the area of a rectangle with a length of 3/7 in. and a width of 2/3 in. below using both a model and a number sentence. Explain where your number sentence is represented in your mode'



The area of the rectangle to the right is 3/7. To get my answer, I used tiling. I split the rectangle into a thirds and then into sevenths. I needed to shade in 2/3 and 3/7 ~~fourths~~ so I found the spots to shade and I shaded them in.

Answer

$\frac{2}{3} \times \frac{3}{7} = \frac{6}{21} = \frac{2}{7}$ Area = $\frac{2}{7}$

Teacher Note: The student clearly and accurately finds the area of a rectangle with fractional sides by tiling with unit squares, and by applying the formula for area (5.NF.4b). The student also explains clearly how the model matches the math demonstrating a deep understanding of multiplication of fractions.

GRADE 5 MATH: TIME FOR RECESS

INSTRUCTIONAL SUPPORTS

The instructional supports on the following pages include a unit outline with formative assessments and suggested learning activities. Teachers may use this unit outline as it is described, integrate parts of it into a currently existing curriculum unit, or use it as a model or checklist for a currently existing unit on a different topic.

Unit Outline

Grade 5 Math: Time for Recess (Multiplying Fractions)

Unit Topic and Length:

- Multiplication of a fraction by a fraction, and other fraction concepts
- 4 weeks, or approximately 15–20 periods of instruction including the initial assessment and the final performance task
- Taught ideally during a Multiplying and Dividing Fractions Unit

Common Core Content Standards:

- **5.NF.3**—Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*
- **5.NF.4a**—Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. Interpret the product $(a/b) \times q$ as a part of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show $(2/3) \times 4 = 8/3$, and create a story context for this equation. Do the same with $(2/3) \times (4/5) = 8/15$. (In general, $(a/b) \times (c/d) = ac/bd$.)*
- **5.NF.4b**—Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.
- **MP.1**—Make sense of problems and persevere in solving them.
- **MP.3**—Construct viable arguments and critique the reasoning of others.
- **MP.6**—Attend to precision.

Big Ideas/Enduring Understandings:

- Fractions are a representation of division of the numerator by the denominator.
- Multiplication of a fraction by a fraction is equivalent to taking a fraction **OF** a fraction.
- Students use the meaning of fractions representing a quantity greater than zero but less than one to understand the concept of multiplication of fractions.

Essential Questions:

- How can I represent a fraction of a fraction using multiplication?
- How can I solve real-world word problems involving division of whole numbers leading to answers in the form of fractions?
- How can I show multiplication of fractions by fractions in multiple ways?

CONTENT:

Interpretations of fractions

- Parts of a whole
- Part of a set
- Measures or quantities
- Simplest form

Parts of a fraction

- Numerator
- Denominator

Connections of fractions

- Context clues
- Strategies to make sense of problems
- Algorithms with fractions, specifically multiplication of fractions
- Relationships between fractions and area of rectangles with fractional sides

SKILLS:

Interpretations of fractions

- **Construct** and **apply** concepts of fractions to reason about real-world situations
- **Connect** concepts of fractions with the symbolic representation, including standard algorithms
- **Construct** pictorial models for fractions (for example, showing 1/4 of 3/8 is 3/32 by drawing an area model)

Parts of a fraction

- **Communicate** and **identify** appropriate mathematical terminologies to refer to fractional parts

Connections of fractions

- **Develop** strategies for multiplying fractions by a fraction
- **Explore** different models to understand the concept of fractions as division of the numerator by the denominator
- **Develop** ways to model situations involving fractions and multiplication of fractions
- **Show** physical models and drawings to help reason about a situation
- **Identify** when multiplication of fractions is the appropriate operation
- **Identify** context clues to help solve a problem

VOCABULARY/KEY TERMS:

- FRACTIONS, FRACTION MODEL, NUMERATOR, DENOMINATOR, EQUIVALENT FRACTIONS, PRODUCT, MULTIPLES, FACTORS, MIXED NUMBERS, INVERSE OPERATION, GRID, MODEL, SET, NUMBER LINE, PATTERNS, HORIZONTAL, VERTICAL, EQUATION, OVERLAP, DIAGRAM, MEASURES, ALGORITHMS, AREA, LENGTH, WIDTH, TILING

ASSESSMENT EVIDENCE AND ACTIVITIES:

INITIAL ASSESSMENT : INVESTIGATION 1

This initial assessment is used to assess student "readiness" for the upcoming unit. Teachers should use student results to find appropriate entry points for students.

FORMATIVE ASSESSMENT: INVESTIGATION 9

The formative assessment, Investigation 9 is used to measure evidence of the priority

standards. This formative assessment should be administered after Exploration 2 before students begin Exploration 3. Teachers should use student results to inform instruction before proceeding to Exploration 3.

FINAL PERFORMANCE TASK: TIME FOR RECESS

This final performance task is used to assess students understanding of fractions as a representation of division of the numerator by the denominator, as well as the use of the standard algorithm and area model for multiplying a fraction by a fraction.

LEARNING PLAN & ACTIVITIES:

- *Note for teachers: This instructional unit was designed to give students a deep understanding of multiplication of fractions. The unit is broken up into three main sections. Exploration 1 gives students a modeling approach with scaffolding to develop a deep conceptualization of multiplying a fraction "of" a fraction. Exploration 2 gives students an opportunity to use their knowledge of modeling multiplication of fractions in area problems with fractional sides. The final step allows students the opportunity to review their models to create mathematical statements that should lead to the standard algorithm for multiplying fractions. In the beginning of this unit, students should be given opportunities to practice fluency in multiplication of whole numbers. As the unit progresses, students should be given opportunities to develop fluency in multiplying with fractions.
- Initial Task: Teachers can use Task 1 as an initial task to measure student "readiness" for the unit. In Grade 4, students worked on adding and subtracting fractions with like-denominators (4.NF.3a, b, c) and multiplying fractions by whole numbers (4.NF.4a, b, c). This work should have included student-created visual fraction models and equations to represent multiplication problems. This previous knowledge, along with the fraction work students should have previously done in Grade 5 in regards to adding and subtracting fractions with unlike denominators, should create the foundation for student learning in this unit. If students have not acquired these prerequisite skills they will need additional support before proceeding with multiplication and division of fractions.
- Review Concept of interpreting a fraction as division of the numerator by the denominator (should be addressed earlier on in the year when students are working on adding and subtracting fractions).
- Opportunities for fluency: As students as building towards "discovery" of the standard algorithm of multiplication of fractions, they should be given time to revisit fluency of multiplication with whole numbers.

- **Exploration 1** (Using models to multiply fractions of a fraction and fractions of a whole): Please see Investigations 1–4 in this unit. Please note,

all tasks are to be done using a model or a diagram ONLY. These investigations are meant to help students develop the concept of a fraction "of" a fraction. As students complete these tasks, teachers should be acting in the role of facilitator. Students should be given time to work through the problem, discuss with a partner their experience and thinking, participate in a student-led "share" time in which classmates explain their thinking, and journal time in which students reflect on their experience. Students can create charted displays of their work. Their classmates can rate their work using a student-friendly rubric.

- **Transitional Activity:** The purpose of this activity is to support students in developing the concept of finding the whole when solving fraction of a fraction multiplication problems. This activity demonstrates how the area model is formed from two individual fraction models and can be used when multiplying fractions.
- **Exploration 2** (Using an area model to multiply fractions by fractions): Please see Investigations 5–8 in this unit. Please note, all tasks are to be done using a model or diagram; however, students can support their thinking using their knowledge of area concepts with whole number sides. Students will use their experience in both Explorations 1 and 2 to develop a strong understanding of the standard algorithm for multiplying fractions. Students should be given time to work through the problem, discuss with a partner their experience and thinking, participate in a student-led "share" time in which classmates explain their thinking, and journal time in which students reflect on their experience.
- **Exploration 3** (Using the models as a springboard for student-led discovery of the multiplication algorithm): Invite students to look back at Investigations 1–8. Ask them to rewrite each situation as a number sentence. In their journals, have students hypothesize how to do the multiplication without a model. This hypothesis is really their first description of the standard algorithm for multiplying fractions. Students should test and revise as needed their hypotheses using Investigation 10. Students should be given time to work through the problem, discuss with a partner their experience and thinking, participate in a student-led "share" time in which classmates explain their thinking, and journal time in which students reflect on their experience. Investigation 11, Part 2 is an additional enrichment activity for students who are more proficient with number line models.
- **Opportunities for fluency:** Students can develop fluency by testing their theories of using the algorithms, revising, and summarizing their hypotheses. Students should be given time to gain fluency in multiplication of fractions.

Additional Support Strategies: Using manipulatives (examples: fraction slips, number lines, graph paper) and carefully scaffolded questions are two strategies that could be helpful for students who need help understanding fractional concepts. For ELL students or students with language barriers, creating a picture-reference guide could be helpful. The use of grid paper in the beginning to help students create models is also another strategy to assist those struggling with fraction concepts.

RESOURCES:

- Van de Walle, John A. *Elementary and Middle School Mathematics: Teaching Developmentally.* Boston, MA: Allyn & Bacon, 2004.
- University of Arizona website for math learning progressions: http://math.arizona.edu/~ime/progressions/

GRADE 5 MATH: TIME FOR RECESS

ADDITIONAL SUPPORTS

Notes on Content for Teachers:

Connection to Grade 4 and earlier Grade 5 content standards:

In Grade 4, students worked on adding and subtracting fractions with like-denominators (4.NF.3a, b, c) and multiplying fractions by whole numbers (4.NF.4a, b, c). This work should have included student-created visual fraction models and equations to represent multiplication problems. This previous knowledge, along with the fraction work students should have previously done in Grade 5 in regards to adding and subtracting fractions with unlike denominators, should create the foundation for student learning in this unit.

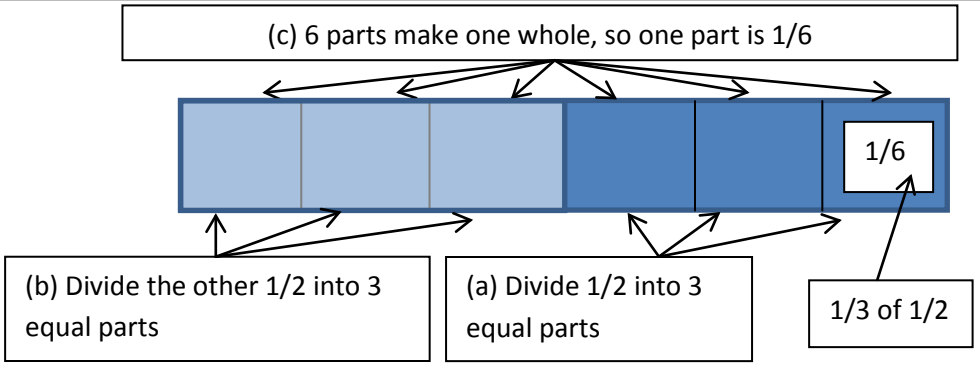
Multiplication as Division (5.NF.3):

In Grade 5, students connect fractions with division, understanding that 5 ÷ 3 = 5/3. They can understand this by thinking about equal sharing, and creating story contexts to represent problems involving division of whole numbers. Students need to use their understanding of fractions to make the connection that 5/3 is one third of 5, which will lead students to the meaning of multiplication by a unit fraction. This naturally leads to multiplication of any quantity by a fraction (5.NF.4a).

Multiplying a fraction or a whole number by a fraction (5.NF.4a):

If students understand that 1/3 X 5 is one part when 5 is partitioned into 3 parts, they can use the same thinking to realize 4/3 X 5 is four parts when 5 is partitioned into 3 parts. This understanding sets students up to understand the general formula for multiplying fractions. They should use various models, fractions strips, and number-lines to support this thinking. This should lead students to the concept of using an area model to find the fraction of a fraction, which leads students to the next standard, finding the area of a rectangle with fractional side lengths (5.NF.4b).

5.NF.4a:
Using a fraction strip to show that
1/3 X 1/2 = 1/6



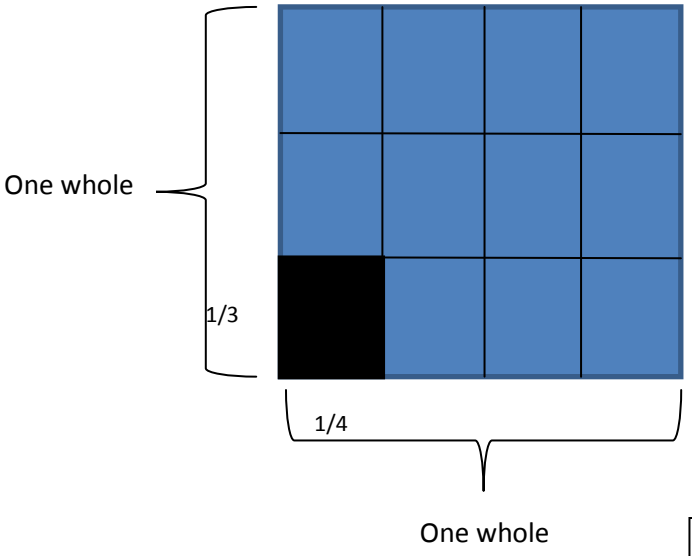
Some problems in this section were taken from: Van de Walle, John A. *Elementary and Middle School Mathematics: Teaching Developmentally.* Boston, MA: Allyn & Bacon, 2004 (pg. 270).

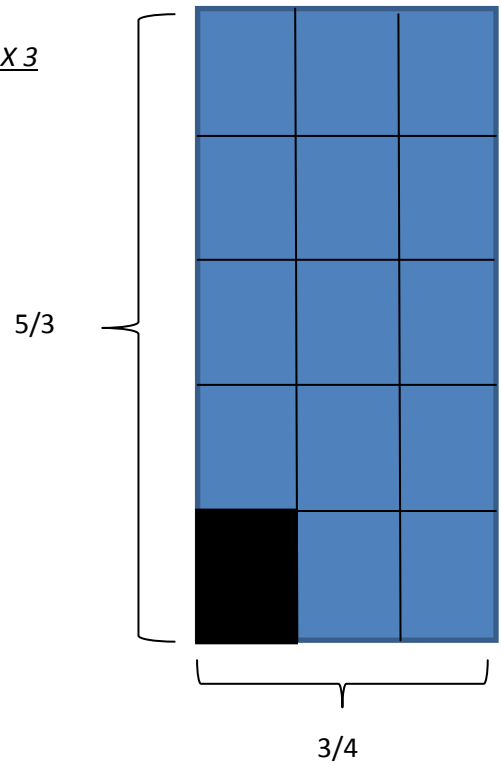
Finding the area of a rectangle with fraction side lengths by tiling it with unit squares and by multiplying the side lengths (5.NF.4b):

Once students make the connection between using area-models to find fractions of a fraction, they can begin to create area-models for rectangles with fraction-sides. Students should see that both situations are multiple representations of multiplication with fraction problems. Making the connection between area-models and their knowledge of the area formula, students should begin to gain a deep understanding of the concept of multiplication of fractions.

Using an area model to show that 3/4 X 5/3 = 3 X 5 / 4 X 3

Because 4 X 3 rectangles 1/4 wide and 1/3 high fit in a unit square, 1/4 X 1/3 = 1/ 4 X 3





The rectangle of width 3/4 and height 5/3 is tiled with 3 X 5 rectangles of area 1/4 X 3, so has area 3 X 5/4 X 3

Some problems in this section were taken from: Van de Walle, John A. *Elementary and Middle School Mathematics: Teaching Developmentally.* Boston, MA: Allyn & Bacon, 2004 (pg. 270).

Time for Recess Explorations:

The following ten explorations are designed for students to work with models to grasp a deep comprehension of multiplying fractions before discovering and applying the standard algorithm. The design is as follows:

- **Exploration 1** (Using models to multiply fractions of a fraction, and fractions of a whole): Please see Investigations 1–4 in this unit. Please note, all tasks are to be done using a model or a diagram ONLY. These investigations are meant to help students develop the concept of a fraction "of" a fraction. As students complete these tasks, teachers should be acting in the role of facilitator. Students should be given time to work through the problem, discuss with a partner their experience and thinking, participate in a student-led "share" time in which classmates explain their thinking, and journal time in which students reflect on their experience. Students can create charted displays of their work. Their classmates can rate their work using a student-friendly rubric.
- **Transitional Activity:** The purpose of this activity is to support students in developing the concept of finding the whole when solving fraction of a fraction multiplication problems. This activity demonstrates how the area model is formed from two individual fraction models and can be used when multiplying fractions.
- **Exploration 2** (Using an area model to multiply fractions by fractions): Please see Investigations 5–9 in this unit. Please note, all tasks are to be done using a model or diagram; however, students can support their thinking using their knowledge of area concepts with whole number sides. Students will use their experience in both Explorations 1 and 2 to develop a strong understanding of the standard algorithm for multiplying fractions. Students should be given time to work through the problem, discuss with a partner their experience and thinking, participate in a student-led "share" time in which classmates explain their thinking, and journal time in which students reflect on their experience.
- **Exploration 3** (Using the models as a spring board for student-led discovery of the multiplication algorithm): Invite students to look back at Investigations 1–9. Ask them to rewrite each situation as a number sentence. In their journals, have students hypothesize how to do the multiplication without a model. This hypothesis is really their first description of the standard algorithm for multiplying fractions. Students should test and revise as needed their hypotheses using Investigations 10 and 11. Students should be given time to work through the problem, discuss with a partner their experience and thinking, participate in a student-led "share" time in which classmates explain their thinking, and journal time in which students reflect on their experience.
- **Opportunities for fluency:** Students can develop fluency by testing their theories of using the algorithms, revising, and summarizing their hypotheses. Students should be given time to gain fluency in multiplication of fractions.

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EXPLORATION 1: Investigation 1

Complete the following problems using only models or diagrams. Be prepared to defend your diagrams with mathematical reasoning:

1. There are 15 cars in Michael's toy car collection. Two-thirds of the cars are red. How many red cars does Michael have?

2. Suzanne has 10 cookies. She wants to share them with her three friends. How many cookies will Suzanne and each of her friends get?

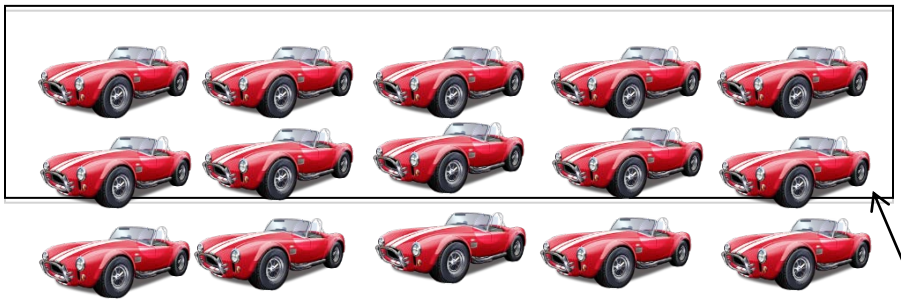
Some problems in this section were taken from: Van de Walle, John A. *Elementary and Middle School Mathematics: Teaching Developmentally.* Boston, MA: Allyn & Bacon, 2004 (pg. 270).

EXPLORATION 1: Investigation 1

Complete the following problems using only models or diagrams. Be prepared to defend your diagrams with mathematical reasoning.

1. There are 15 cars in Michael's toy car collection. Two-thirds of the cars are red. How many red cars does Michael have?

Answer = 10 Cars

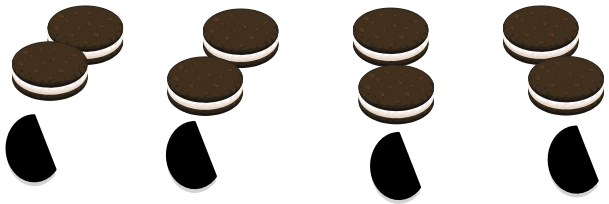


Teacher Note: This is one possible model. Many different diagrams are possible, and as long as they match student thinking they should be accepted.

2. Suzanne has 10 cookies. She wants to share them with her three friends. How many cookies will Suzanne and each of her friends get?

Answer =

2 1/2 cookies



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EXPLORATION 1: Investigation 2

Complete the following problems using only models or diagrams. Be prepared to defend your diagrams with mathematical reasoning.

1. Wayne filled 5 glasses with 2/3 liter of soda in each glass. How much soda did Wayne use?

2. Linda's mom has 8 cartons of eggs. Each of the egg cartons is 3/4 full. If Linda's mom wanted to rearrange the eggs into full cartons, how many egg cartons would Linda's mom have?

Some problems in this section were taken from: Van de Walle, John A. *Elementary and Middle School Mathematics: Teaching Developmentally.* Boston, MA: Allyn & Bacon, 2004 (pg. 270).

EXPLORATION 1: Investigation 2

Complete the following problems using only models or diagrams. Be prepared to defend your diagrams with mathematical reasoning.

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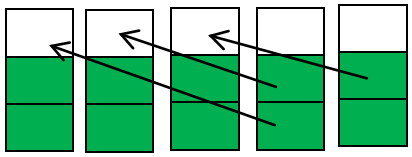
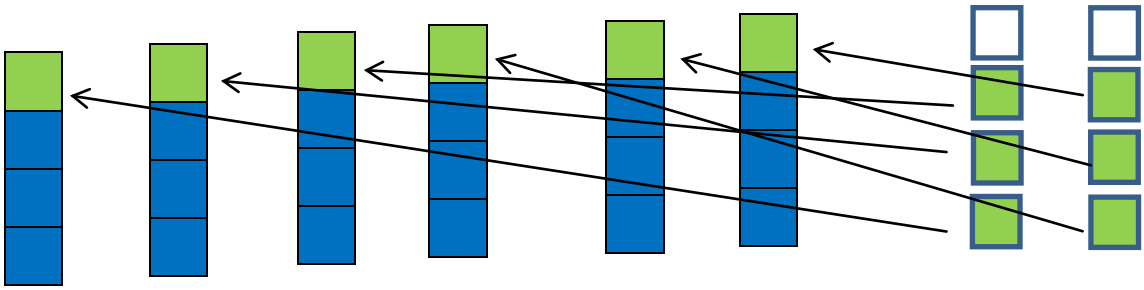


Answer = 3 1/3 liters

Teacher Note: This is one example of a model; however, students will show this various ways. Credit should be given to all accurate and clear diagrams.

2. Linda's mom has 8 cartons of eggs. Each of the egg cartons is 3/4 full. If Linda's mom wanted to rearrange the eggs into full cartons, how many egg cartons would Linda's mom have?

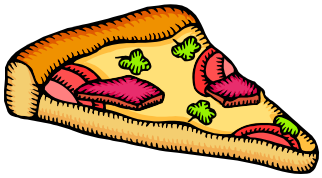


Answer = 6 full cartons of eggs

Some problems in this section were taken from: Van de Walle, John A. *Elementary and Middle School Mathematics: Teaching Developmentally.* Boston, MA: Allyn & Bacon, 2004 (pg. 270).

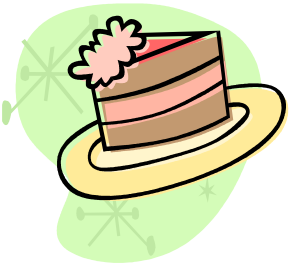
EXPLORATION 1: Investigation 3

Complete the following problems using only models or diagrams. Be prepared to defend your diagrams with mathematical reasoning.



1. You have 3/4 of a pizza left. If you give 1/3 of the leftover pizza to your brother, how much of a whole pizza will your brother get?

2. Someone ate 1/10 of the cake. If you eat 2/3 of the cake that is left, how much of a whole cake will you have eaten?



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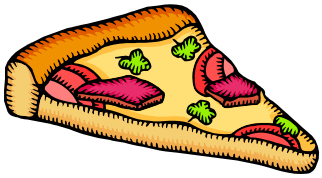
EXPLORATION 1: Investigation 3

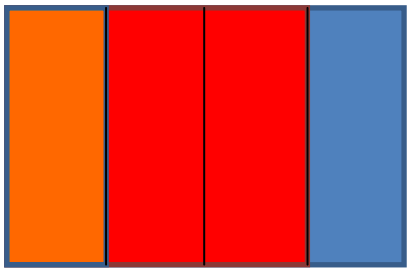
Complete the following problems using only models or diagrams. Be prepared to defend your diagrams with mathematical reasoning.



1. You have 3/4 of a pizza left. If you give 1/3 of the leftover pizza to your brother, how much of a whole pizza will your brother get?

3/4 of the pizza left was colored red, and the 1/4 pizza gone was colored blue.

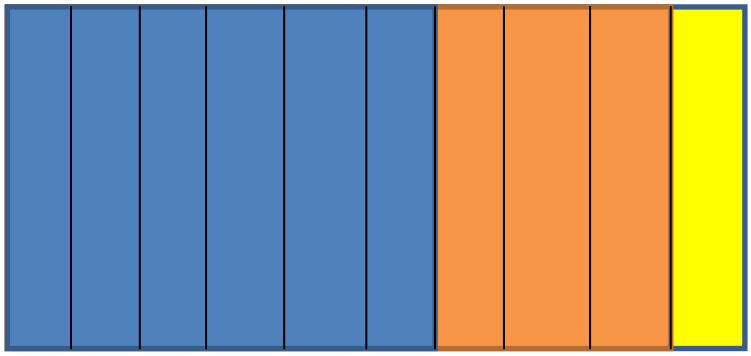
The 1/3 of the remaining pizza (1/4 of the original) was colored orange.



Answer = 1/4 of the entire pizza

2. Someone ate 1/10 of the cake. If you eat 2/3 of the cake that is left, how much of a whole cake will you have eaten?

Answer = 3/5 of the cake



The yellow represents the 1/10 already eaten. The orange represents the 1/3 left after you eat 2/3 of the remaining cake. The blue represents the 2/3 of the remaining cake (6/10 or 3/5 of the original cake) that was eaten.

Notice for these problems that the student does not have to break the rectangle into smaller pieces; they can identify the fractions by the original denominator.

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EXPLORATION 1: Investigation 4

Complete the following problem using only models or diagrams. Be prepared to defend your diagrams with mathematical reasoning.

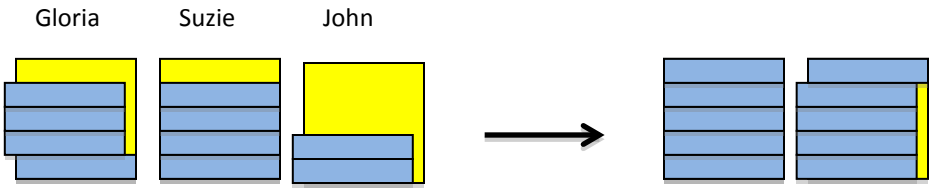
1. At lunch Gloria ate 4/5 of a bag of potato chips, Suzie ate 4/5 of a bag of potato chips, and John ate 1/2 of what Suzie ate. How many bags of potato chips did they eat in all?

Some problems in this section were taken from: Van de Walle, John A. *Elementary and Middle School Mathematics: Teaching Developmentally.* Boston, MA: Allyn & Bacon, 2004 (pg. 270).

EXPLORATION 1: Investigation 4

Complete the following problem using only models or diagrams. Be prepared to defend your diagrams with mathematical reasoning.

1. At lunch Gloria ate 4/5 of a bag of potato chips, Suzie ate 4/5 of a bag of potato chips, and John ate 1/2 of what Suzie ate. How many bags of potato chips did they eat in all?



4/5 + 4/5 + 1/2(4/5) = 10/5

Teacher Note: Two possible solutions are shown. The model above has the student drawing the 4/5 bag of chips for Gloria and Suzie and the 2/5 bag of chips for John, then moving the 2/5 from John's bag of chips to determine that all together they ate 10/5 or 2 bags of chips. The second solution is the addition algorithm for solving this problem. The algorithm includes 1/2 of 4/5, which equals 2/5.

Some problems in this section were taken from: Van de Walle, John A. *Elementary and Middle School Mathematics: Teaching Developmentally.* Boston, MA: Allyn & Bacon, 2004 (pg. 270).

Transitional Problem:

Complete the following problem using only models or diagrams. Be prepared to defend your diagrams with mathematical reasoning.

1. Sophie's friend gave her 1/2 of a chocolate bar. She ate 1/5 of it. How much did she eat?

Some problems in this section were taken from: Van de Walle, John A. *Elementary and Middle School Mathematics: Teaching Developmentally.* Boston, MA: Allyn & Bacon, 2004 (pg. 270).

Transitional Problem:

Complete the following problem using only models or diagrams. Be prepared to defend your diagrams with mathematical reasoning.

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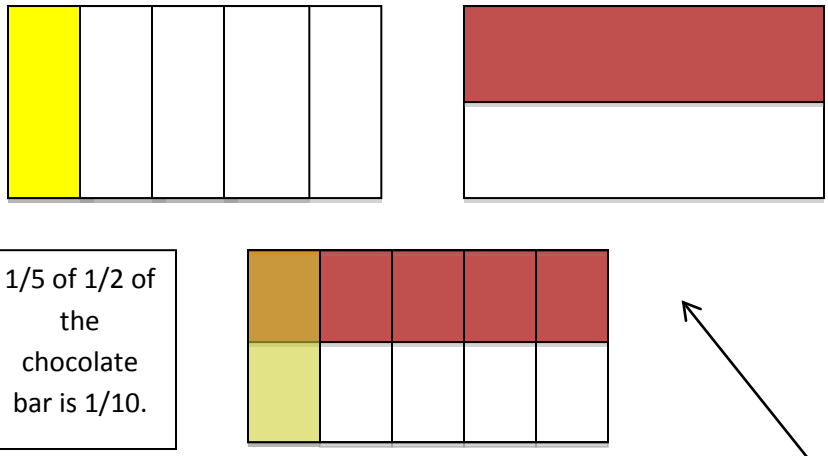


1/5 of 1/2 of the chocolate bar is 1/10.

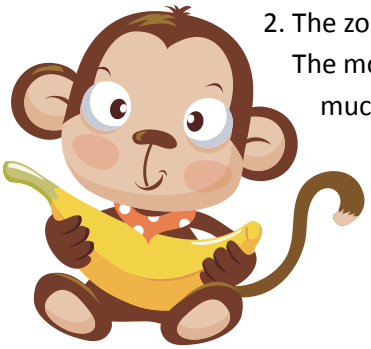
Teacher Note: The models on this page are examples of how area models are developed from two separate fraction models in multiplication. By representing 1/5 in one model and 1/2 in the other, students can visualize how an area and array model is formed and how 1/10 can be determined by looking at the overlapping area. This is the type of model students should be using as they transition to finding the area of rectangles with fractional sides. These models can be used to support students who may have difficulty creating fractional models for multiplication.

Some problems in this section were taken from: Van de Walle, John A. *Elementary and Middle School Mathematics: Teaching Developmentally.* Boston, MA: Allyn & Bacon, 2004 (pg. 270).

EXPLORATION 2: Investigation 5

Complete the following problems using only models or diagrams. Defend your models/diagrams with mathematical reasoning.

1. Zack had 2/3 of the lawn left to cut. After lunch, he cut 3/4 of the grass he had left. How much of the whole lawn did Zack cut after lunch?



2. The zookeeper had a huge bottle of the animals' favorite liquid treat, Zoo Cola. The monkey drank 1/5 of the bottle. The zebra drank 2/3 of what was left. How much of the bottle of Zoo Cola did the zebra drink?

Some problems in this section were taken from: Van de Walle, John A. *Elementary and Middle School Mathematics: Teaching Developmentally.* Boston, MA: Allyn & Bacon, 2004 (pg. 270).

EXPLORATION 2: Investigation 5

Complete the following problems using only models or diagrams. Defend your models/diagrams with mathematical reasoning.

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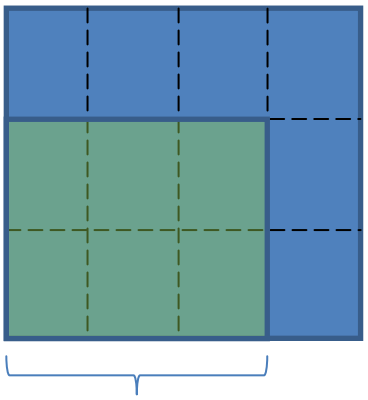
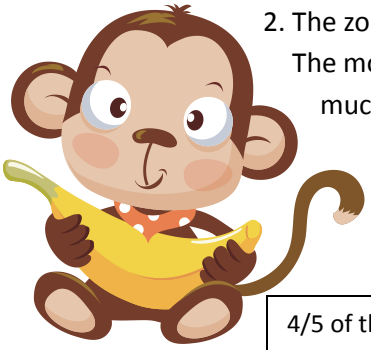


2/3 of the yard left to cut

3/4 of the 2/3 of grass he cuts after lunch

Teacher Note: The models on this page are examples of area models for multiplication of fractions. This is the type of model students should be working towards as they transition to finding the area of rectangles with fractional sides.

Answer = 6/12 or 1/2 of the yard



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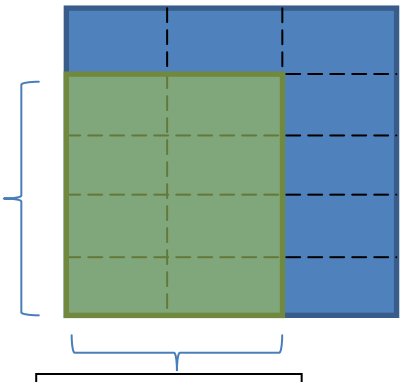


4/5 of the bottle remaining after the monkey drank 1/5

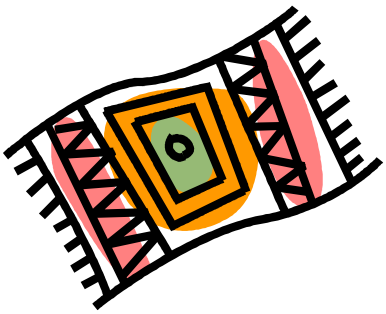
2/3 of the remaining 4/5 that the zebra drank

Answer = 8/15 of the bottle

Some problems in this section were taken from: Van de Walle, John A. *Elementary and Middle School Mathematics: Teaching Developmentally.* Boston, MA: Allyn & Bacon, 2004 (pg. 270).

EXPLORATION 2: Investigation 6

Complete the following problems using only models or diagrams. Defend your models/diagrams with mathematical reasoning and your understanding of area.

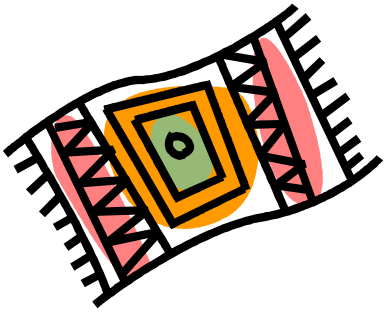


1. Zack's classroom has a carpet that is 2/3 of the classroom long and 3/4 of the classroom wide. How much of the classroom area does the carpet take up?

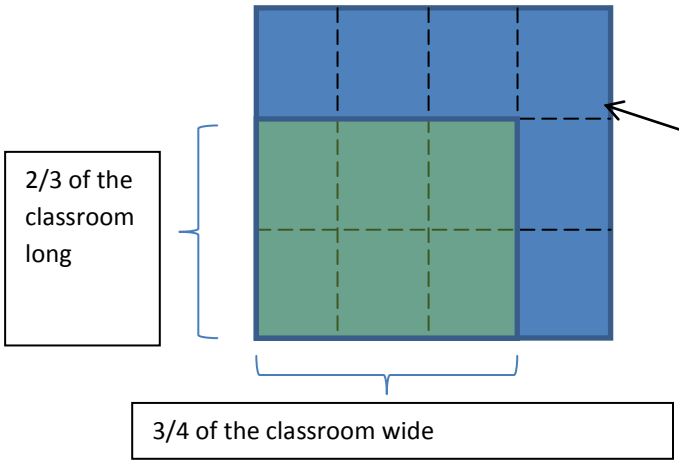
Some problems in this section were taken from: Van de Walle, John A. *Elementary and Middle School Mathematics: Teaching Developmentally.* Boston, MA: Allyn & Bacon, 2004 (pg. 270).

EXPLORATION 2: Investigation 6

Complete the following problems using only models or diagrams. Defend your models/diagrams with mathematical reasoning and your understanding of area.



1. Zack's classroom has a carpet that is 2/3 of the classroom long and 3/4 of the classroom wide. How much of the classroom area does the carpet take up?



Teacher Note: This area model is exactly the same as the one from Problem 1 in Investigation 5. The idea is that students will recognize that finding a fraction of a fraction is mathematically equivalent to finding the area of rectangle with fractional side lengths. Because students already know the area formula uses the operation of multiplication, this should lead students to the conclusion that when you are finding a fraction of a fraction the operation you are using is multiplication.

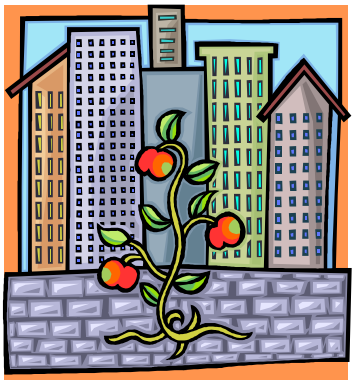
Answer = 6/12 or 1/2 of the classroom area

Some problems in this section were taken from: Van de Walle, John A. *Elementary and Middle School Mathematics: Teaching Developmentally.* Boston, MA: Allyn & Bacon, 2004 (pg. 270).

EXPLORATION 2: Investigation 7

Complete the following problems using only models or diagrams. Defend your models/diagrams with mathematical reasoning and your understanding of area.

1. Emily wants to plant a vegetable garden in her backyard. Emily's mom says she can make a garden that has a width of 5/6 of the yard and a length of 1/4 of the yard. What is the area of Emily's new garden?



Some problems in this section were taken from: Van de Walle, John A. *Elementary and Middle School Mathematics: Teaching Developmentally.* Boston, MA: Allyn & Bacon, 2004 (pg. 270).

EXPLORATION 2: Investigation 7

Complete the following problems using only models or diagrams. Defend your models/diagrams with mathematical reasoning and your understanding of area.

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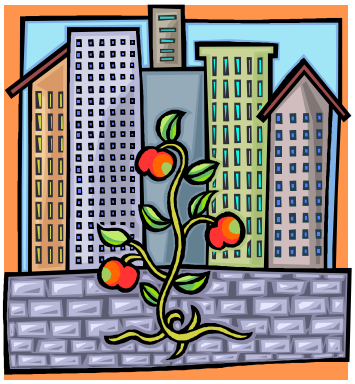
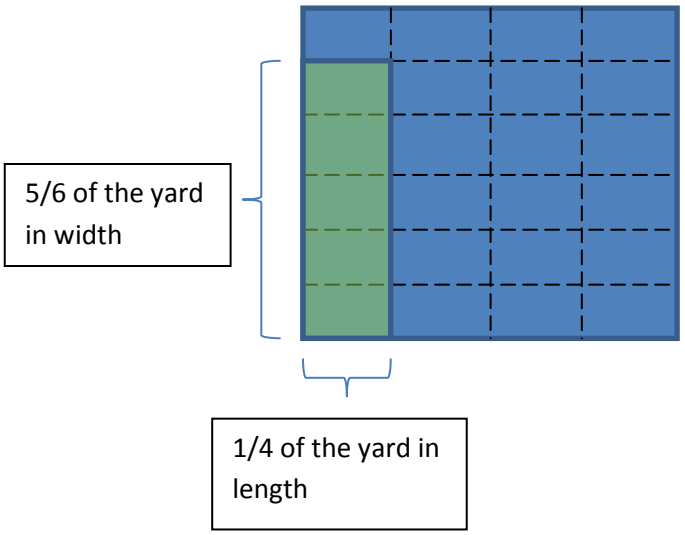




A = L X W

A = 5/6 X 1/4

A = 5/24

Teacher Note: The model used here is an accurate model that uses tiling to find the area of a rectangle with fractional sides. There are multiple possibilities for this model, but students should only receive full credit if they use an accurate area model. The appropriate use of the area formula that matches the model is here as well.

Some problems in this section were taken from: Van de Walle, John A. *Elementary and Middle School Mathematics: Teaching Developmentally.* Boston, MA: Allyn & Bacon, 2004 (pg. 270).

EXPLORATION 2: Investigation 8

Complete the following problems using only models or diagrams. Defend your models/diagrams with mathematical reasoning and your understanding of area.

1. Erik and Anthony are big Lego fans. Each of them decides to set up an area in the room that is only to be used for Lego building. Erik uses a rectangular area that is 7/8 yd. by 3/5 yd. Anthony creates a rectangular area that is 3/4 yd. by 4/5 yd. Anthony says he uses a bigger area for Lego building than Erik. Is he correct? Why or why not?

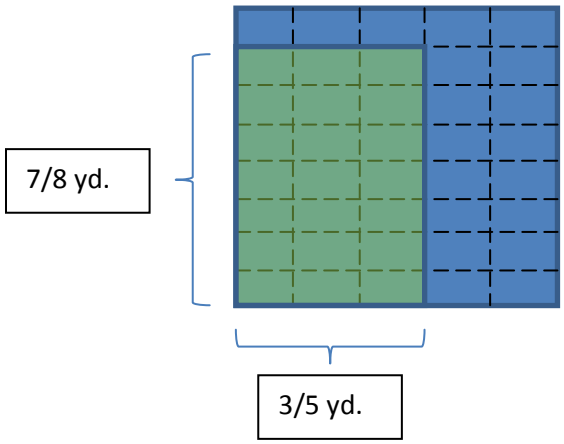
Some problems in this section were taken from: Van de Walle, John A. *Elementary and Middle School Mathematics: Teaching Developmentally.* Boston, MA: Allyn & Bacon, 2004 (pg. 270).

EXPLORATION 2: Investigation 8

Complete the following problems using only models or diagrams. Defend your models/diagrams with mathematical reasoning and your understanding of area.

1. Erik and Anthony are big Lego fans. Each of them decides to set up an area in the room that is only to be used for Lego building. Erik uses a rectangular area that is 7/8 yd. by 3/5 yd. Anthony creates a rectangular area that is 3/4 yd. by 4/5 yd. Anthony says he uses a bigger area for Lego building than Erik. Is he correct? Why or why not?

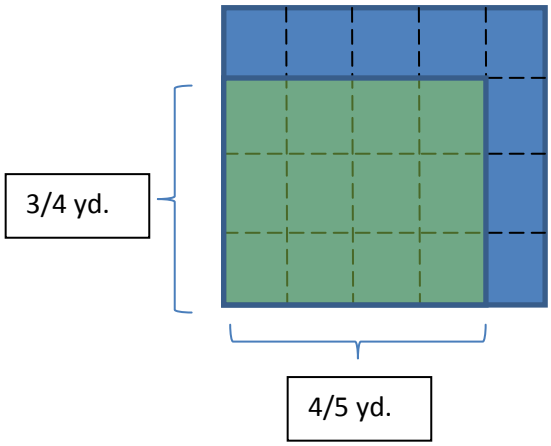
Erik's Lego Area



A = L X W
A = 7/8 yd. X 3/5 yd.
A = 21/40 sq. yd.

Explanation: Anthony is correct. The area of Anthony's room is 12/20 or 24/40 sq. yd. The area of Erik's room is 21/40 sq. yd. 24/40 sq. yd. > 21/40 sq. yd., so Anthony is correct when saying his Lego area is bigger.

Anthony's Lego Area



A = L X W
A = 3/4 yd. X 4/5 yd.
A = 12/20 sq. yd.
12/20 = 24/40

Teacher Note: In order to receive full credit, an accurate area model, matching use of the area formula, and a clear explanation that concludes in Anthony's reasoning being correct must both be included.

Some problems in this section were taken from: Van de Walle, John A. *Elementary and Middle School Mathematics: Teaching Developmentally.* Boston, MA: Allyn & Bacon, 2004 (pg. 270).

FORMATIVE ASSESSMENT: Investigation 9

Complete the following problems using only models or diagrams. Defend your models/diagrams with mathematical reasoning and your understanding of area.

1. Daniella wants to buy a carpet for her room. She only has $50 to spend. She wants to cover as much of her room as possible. She finds two carpets that she can afford. Carpet A measures 2/3 of her room in length and 3/5 of her room in width. Carpet B measures 1/3 of her room in length and 3/4 of her room in width. Which carpet should she buy and why?

Some problems in this section were taken from: Van de Walle, John A. *Elementary and Middle School Mathematics: Teaching Developmentally.* Boston, MA: Allyn & Bacon, 2004 (pg. 270).

FORMATIVE ASSESSMENT: Investigation 9

Complete the following problems using only models or diagrams. Defend your models/diagrams with mathematical reasoning and your understanding of area.

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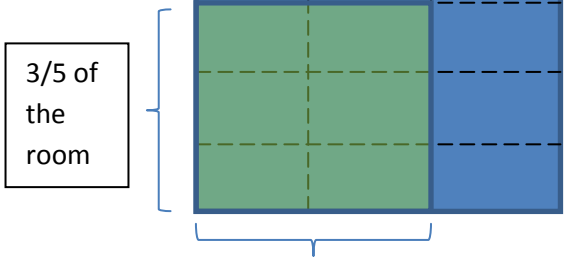
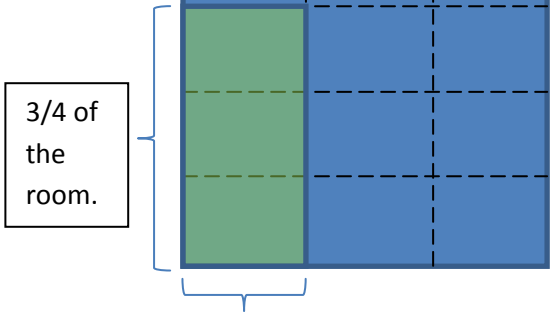
Carpet A



A = L X W
A = 3/5 X 2/3
A = 6/15 or 2/5 of the room's area

Carpet B



A = L X W
A = 3/4 X 1/3
A = 3/12 or 1/4 of the room's area

Explanation: Daniella should buy Carpet A. The area of Carpet A is 6/15 or 2/5 of the room. The area of Carpet B is 3/12 or 1/4 of the room. Using a common denominator, I know that 2/5 is equal to 8/20 and 1/4 is equal to 5/20. Because 8/20 > 5/20, I know that Carpet A is the bigger carpet.

Teacher Note: In order to receive full credit, an accurate area model, matching use of the area formula, and a clear explanation that concludes in Anthony's reasoning being correct must all be included.

Some problems in this section were taken from: Van de Walle, John A. *Elementary and Middle School Mathematics: Teaching Developmentally.* Boston, MA: Allyn & Bacon, 2004 (pg. 270).

EXPLORATION 3: Investigation 10

Complete the following problems using the algorithm you developed for multiplying fractions. Use models/diagrams to support your mathematical equation.

1. Sharon has a box of pencils. She uses 3/7 of the box during the week. She gives 1/2 of the pencils remaining to her friend. What fraction of the box does Sharon have left?

2. Phillip wants to decorate one of the walls in his room with posters of his favorite sports stars. A bookcase already takes up 1/6 of the room. If Phillip wants to decorate 2/3 of the remaining area with posters, what fraction of the entire wall will he be decorating?

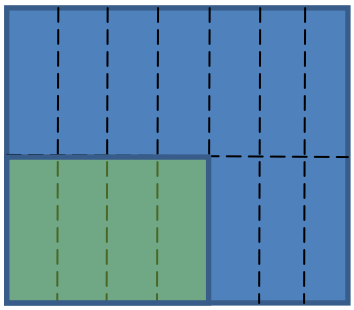
Some problems in this section were taken from: Van de Walle, John A. *Elementary and Middle School Mathematics: Teaching Developmentally.* Boston, MA: Allyn & Bacon, 2004 (pg. 270).

EXPLORATION 3: Investigation 10

Complete the following problems using the algorithm you developed for multiplying fractions. Use models/diagrams to support your mathematical equation.

1. Sharon has a box of pencils. She uses 3/7 of the box during the week. She gives 1/2 of the pencils remaining to her friend. What fraction of the box does Sharon have left?

7/7 – 3/7 = 4/7
4/7 X 1/2 = 4/14
4/14 = 2/7
Answer: Sharon has 2/7 of the box left.

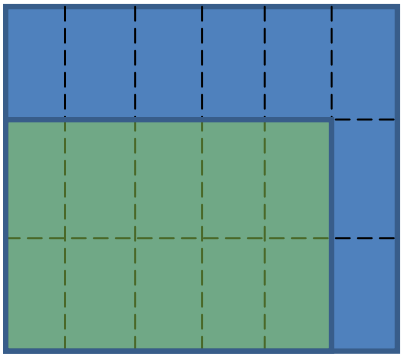


4/7 remaining after Sharon uses 3/7

1/2 of the box left after she gives 1/2 to her friend

Teacher Note: The model here is an example of an area model for finding the fraction of a fraction. Models at this point should be accurately labeled and match the algorithm used.

2. Phillip wants to decorate one of the walls in his room with posters of his favorite sports stars. A bookcase already takes up 1/6 of the room. If Phillip wants to decorate 2/3 of the remaining area with posters, what fraction of the entire wall will he be decorating?



2/3 of the remaining area of the wall

5/6 of the wall left after accounting for the 1/6 the bookcase takes up

6/6 – 1/6 = 5/6
5/6 X 2/3 = 10/18
10/18 = 5/9
Answer: Phillip will be decorating 5/9 of the wall.

Some problems in this section were taken from: Van de Walle, John A. *Elementary and Middle School Mathematics: Teaching Developmentally.* Boston, MA: Allyn & Bacon, 2004 (pg. 270).

EXPLORATION 3: Investigation 11

Complete the following problem using tiling and the area formula.

1. Please find the area of a rectangle with a length of 3/8 in. and a width of 1/4 in.

ENRICHMENT ACTIVITY

Complete the following problem using a number line.

Create a number line for multiples of the number 6. Please indicate the product above the number line for the factor on the number line. Use the following numbers: 0, 2/5, 1/2, 2/3, 1, 2, 3, 4.

Some problems in this section were taken from: Van de Walle, John A. *Elementary and Middle School Mathematics: Teaching Developmentally.* Boston, MA: Allyn & Bacon, 2004 (pg. 270).

EXPLORATION 3: Investigation 11

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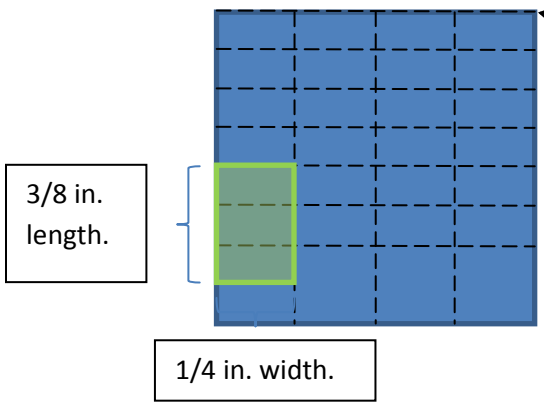


A = L X W

A = 3/8 X 1/4

A = 3/32

Answer = 3/32 sq. in.

The area model on the left shows the student tiling a rectangle with a length of 3/8 in. and a width of 1/4 in. using 1/32 in. squares. The area is 3/32 sq. in.

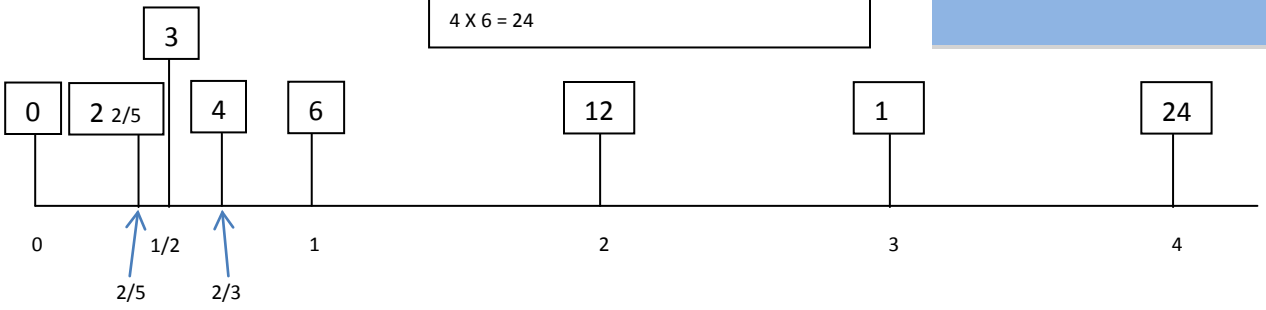
The use of the area formula is clearly indicated as well. The student writes the formula and uses the correct numbers. They follow the algorithm correctly and obtain an answer that matches their model.

ENRICHMENT ACTIVITY

Complete the following problem using a number line.

Create a number line for multiples of the number 6. Please indicate the product above the number line for the factor on the number line. Use the following numbers: 0, 2/5, 1/2, 2/3, 1, 2, 3, 4.

Math Work:
0 X 6 = 0
2/5 X 6 = 12/5 = 2 2/5
1/2 X 6 = 6/2 = 3
2/3 X 6 = 12/3 =4
1 X 6 = 6
2 X 6 = 12
3 X 6 = 18
4 X 6 = 24



Teacher Note: To get the students to complete a number-line like the one above will take some modeling, but it's a great way for students to make some big connections.

Some problems in this section were taken from: Van de Walle, John A. *Elementary and Middle School Mathematics: Teaching Developmentally.* Boston, MA: Allyn & Bacon, 2004 (pg. 270).

Cheat-Sheet with Suggestions for Helping Students Access Information

Barrier to Learning	Suggested Strategy
Student lacks understanding of math language	Review domain specific vocabulary; create a picture dictionary with student.
Student lacks basic fractional knowledge	Review prior lessons in fractions; use manipulatives to explore fraction concepts.
Students have problems in understanding math word problems (reading comprehension)	Use key words and "picture stories" to help students identify the appropriate operation.
	Build vocabulary through repeated classroom use and picture dictionary.
	Work on reading and understanding problems through modeling in small groups and peer-to-peer situations.
Student struggles with multi-step problems	Break the problem into smaller tasks, an understandable sequence.
Student struggles with writing explanations and math reasoning	Continued use of math journaling and share time in which classmates critique each other can help strengthen this.
Student struggles with creating a fractional model for multiplying fractions	Use manipulatives such as folding a piece of paper vertically and horizontally to find the fraction of a fraction.
Student misunderstands the concept of multiplication of a fraction resulting in a product less than what you started with	Use number lines to show that a fraction falls between 0 and 1; and then make the connection for students that the product, when multiplying by a fraction, must fall between zero and the original number.

Some problems in this section were taken from: Van de Walle, John A. *Elementary and Middle School Mathematics: Teaching Developmentally.* Boston, MA: Allyn & Bacon, 2004 (pg. 270).

Suggested Universal Design for Learning Supports:

Guideline:	Suggestion:
1: Provide options for perception	Use graph paper with colored pencils to practice multiplying fractions.
	Explain and show various models and diagrams.
	Use read-alouds and/or videos that include the concept of multiplying fractions.
2: Provide options for language, mathematical expressions, and symbols	Create student-specific word-walls that include symbols to promote understanding of math terms.
	Use models and diagrams to illustrate expressions and symbols.
3: Provide options for comprehension	Review prior fractional knowledge to relate to basic fraction concepts to multiplying of fractions.
	Identify the patterns and relationships associated with multiplication of fractions.
4: Provide options for physical action	Explore fractional pieces as manipulatives, and allow students the opportunity to use them when attempting investigations.
	Play games that allow for practice of multiplying fractions.
	Use technology-related manipulatives through use of a SMART board or other device.
5: Provide options for expression and communication	Allow students to work in partnerships when attempting the investigations.
	Use a share-time to give students an opportunity to explain their math thinking and critique the reasoning of classmates.
	Use math journals as an avenue for daily student self-reflection.
	Provide opportunities for students to use technology for responses.
6: Provide options for executive function	Design parallel tasks so all students have an opportunity to engage on same concept.
	Identify individual entry-points on assessments.
7: Provide options for recruiting interest	Use real-world problems that allow students to see application.
	Allow students to create their own problems.
	Allow choice of various problems and/or ways to solve them.
8: Provide options for sustaining effort and persistence	Allow time for student to work individually at first, then share with a partner.
	Provide various levels of problems for challenge.
	Provide teacher feedback in the form of questioning that pushes students' thinking.
9: Provide options for self-regulation	Use rubrics for student reflection.
	Allow multiple opportunities for student choice (i.e., manipulative use, strategy to solve, etc.)

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ENRICHMENT ACTIVITIES

For students who demonstrate mastery of content and practice standards at a rate quicker than others, the following tic-tac-toe choice board can be used as a form of differentiation. Students can choose three activities in a row (vertically, horizontally, or diagonally) and complete the activities. Student work should be stapled to the tic-tac-toe board and submitted to the teacher for evaluation. Each activity is designed to help students further conceptualize and deepen understanding of multiplication of fraction concepts.

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Name:_____ **Date:**_____

MULTIPLYING FRACTIONS TIC-TAC-TOE CHOICE BOARD

It's time for some tic-tac-toe! Challenge yourself by completing an activity from the tic-tac-toe board below. If you complete three activities in a row (vertically, horizontally, or diagonally), turn in this sheet along with any work stapled to it and receive special recognition in our classroom!

Make a Picture Dictionary Create a picture dictionary with at least 15 math vocabulary terms that students come across when multiplying fractions. Each term should have a definition and diagram associated with it.	**Make a Movie** Use a flip-cam or other recording device to make a 5–10 minute video clip of yourself teaching others how to multiply fractions. Be sure to use models, number sentences, and clear explanations in your video.	**Create a Children's Book** Create a children's book which explains clearly for younger readers how you can use models to multiply fractions. Make sure to include words, diagrams, and illustrations that would grab a younger reader's attention.
Create a Quiz Create a short quiz on multiplying fractions. Your quiz should have at least 10 questions and consist of word problems as well as number problems. Have a classmate take the quiz and give them a grade.	**Design Fraction Multiplication Flashcards** Create a set of flash cards for multiplying fractions. On one side have the number sentence, on the other side state the product and include a model.	**Rewrite a Recipe** Find a recipe in a cookbook and rewrite the measurements for the list of ingredients as multiplication problems. Ex. 1/3 cup of water could be rewritten as 1/2 X 2/3 cup of water.
Write and Perform a Song Write a song that students can learn to help them multiply fractions. Perform the song live or record it on a video and play it for the class.	**Make a Board Game** Create a board game that students need to multiply fractions to play. Make sure you create the board, cards, pieces, and clear directions for your game.	**Write a Newspaper** Create a newspaper that has articles about multiplying fractions. Be creative and imaginative, but make sure that each article incorporates multiplication of fractions in one way or another. Feel free to include comics, illustrations, and pictures.

I have completed, reviewed, and revised at least three activities crossed out above and work is stapled to this page.

Student signature Date

Some problems in this section were taken from: Van de Walle, John A. *Elementary and Middle School Mathematics: Teaching Developmentally.* Boston, MA: Allyn & Bacon, 2004 (pg. 270).